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STUDY OF GRAVITY-DRIVEN FILM FLOW WITH VARIABLE PHYSICAL PROPERTIES ALONG AN INCLINED WALL

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Abstract-This paper deals with the gravity-driven Newtonian, laminar film flow along an inclined wall with variable physical properties. The similarity transformation inspired by Howarth-Dorodnitsyn transformation has been used here since Falkner-Skan type of similarity transformations is no longer applicable for variable physical properties. Variation of thermo -physical properties with temperature for water, air and engine oil has been observed. Some empirical correlations for the properties are used for the dependency of temperature on properties. Although typically this type of flow is solved as initial value problem with shooting technique, in this thesis, rather it is solved as a non-linear two point boundary value problem more easily. The ordinary differential equations are recast as a first order ordinary differential equations, and then solved by a function 'bvp4c' in MATLAB. Although, there exists variations of properties with temperature for air, water and engine oil, which clearly observed in the corresponding results, results are investigated only for water knowing that other fluids qualitatively will give similar behavior.

Index terms- variable physical properties, gravity driven Newtonian flow, boundary value problem

1 INTRODUCTION

Gravity-driven flow of liquid in thin film along a solid surface is a common phenomenon which occurs in everyday life as well as in a variety of industrial applications, for instance, in various types of heat and mass transfer equipments like coolers, evaporators, trickling filters Andersson[1], and in chemical and nuclear reactors [2]. The boundary layer equations for this type of flow, developed first by Prandtl [3], are studied for a long time based on the Boussinesq approximation. The temperature dependency of other molecular transport properties (viscosity and thermal conductivity), and thermal conductivity is neglected in the governing partial equations of the boundary layer. This approximation is suited well for the case of small temperature differences between the wall and the ambient fluid.

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However, there are numerous practical applications in various branches of industry, for example, metallurgical, chemical, mechanical, and food industries [4], where heat transfer in boundary layers and film flows caused by acceleration often involves large temperature differences. In this case, not only the density but also other physical properties, such as, viscosity, thermal conductivity, and specific heat also vary with this large temperature difference. A number of experimental investigations [5] exist for these variations of the properties with temperature, and subsequently, several correlations are proposed by the researchers [6]. Therefore, it is important to study the hydrodynamic and thermal characteristics of falling film flow with larger temperature differences.

The hydrodynamics of gravity-driven film flow and the accompanying heat and mass transfer have been extensively studied over the years both theoretically and experimentally. However, only the studies pertaining to the current problem is discussed here. The earliest theoretical consideration of variable thermo-physical properties for free convection is the perturbation analysis of Hara [7] for air. The solution is applicable for small values of the perturbation parameter, $\varepsilon H = (T_w - T_\infty)/T_\infty$. Tataev [8] also investigated the free convection of a gas with variable viscosity. A well-known analysis of the variable fluid property problem for laminar free convection on an isothermal vertical flat plate has been presented by Sparrow and Gregg [9]. They considered five different gases and provided the corresponding solutions of the boundary layer equations. Gray and Giogini [10] discussed the validity of the Boussinesq approximation and proposed a method for analyzing natural convection flow with fluid properties assumed to be a linear function of the temperature and pressure.

Mashelkar and Chavan [11] provided a more general solution of this problem. Van der Mast et al. [12] indicated that for accelerating film flow the heat transfer coefficient for the inlet section considerably higher than further downstream.

Pop, Watanabe and Komishi [13] on the steady of laminar gravity-driven film flow along a vertical wall for Newtonian fluids, investigated for the effect of injection/suction on the heat transfer, which is based on Falkner–Skan type transformation. However, for variable thermo-physical properties the Falkner-Skan type of transformation is no longer applicable. Later, Andersson et al. [2] introduced a new similarity transformation for the solution of vertical falling film with variable properties.

The hydrodynamics of gravity-driven power-law films has been studied theoretically by means of the integral method approach and similarity analysis.

A variety of solution methods can be applied to investigate the hydrodynamics of gravity-driven film flow problem, for example, integral method approach [14–16], similarity analysis [17,18], initial-value problem Andersson et al. [2]. Initial-value methods are typically solved by using either the finite-difference or Runge-Kutta method, combined with a shooting technique. It seems that there is no research work, to our knowledge, which solves the fluid flow and heat transfer of falling film with variable properties by the boundary value problem, which is more convenient since no corresponding adjoint equations or shooting is necessary. In this paper, thus, coupled governing ordinary differential equations are solved more easily as a two-point boundary value problem.

To demonstrate the effect of variable physical properties on the flow and heat transfer, nonlinear empirical temperature correlations for water are used. Results for a film with inflow temperature 20 °C will be presented and compared with results for a film at 60 °C.

2 MATHEMATICAL FORMULATION

Consider a laminar, incompressible, two-dimensional liquid film flowing down along a vertical wall under gravitational force, as shown in Fig. 1. The Cartesian coordinates (x,y) are chosen, where x is in the streamwise direction (i.e. falling film direction), and y is in the direction normal to the flow. The film density, the dynamic viscosity, the thermal conductivity and the specific heat are denoted by ρ , μ , κ and C_p , respectively. The wave formation in the film free surface is not considered. A free stream velocity is U which is the function of x. A uniform flow with zero velocity i.e. U (x = 0) = 0, enters the system with a constant temperature T_0 and falls vertically down along the smooth wall which is kept at a constant temperature T_w. A hydrodynamic and a thermal boundary layer are developed along the vertical wall with thicknesses $\delta(x)$ and $\delta_T(x)$, respectively. The flow outside

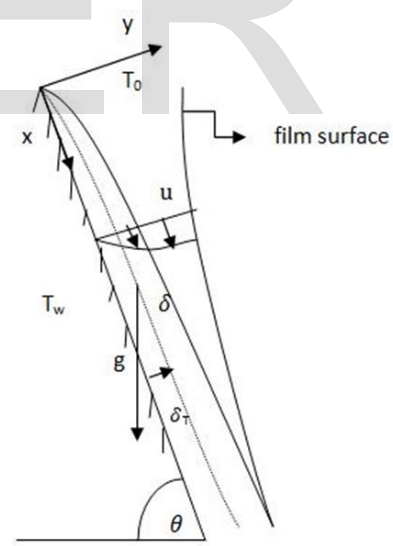

Fig.1.Schematic representation of gravity driven film flow and coordinate system

the hydrodynamic boundary layer remains quasi-one-dimensional and is governed by the inviscid equation of motion as follows,

$$\frac{\rho u dx}{dx} = \rho g \sin\theta \qquad (1)$$

where g is the gravitational acceleration. Equation (1)is readily integrated once with the condition zero velocity at the inlet to give,

$$U(x) = (2gx)^{1/2}. \qquad (2)$$

The part of the liquid film outside the thermal boundary layer is unaffected by the heat transfer between the vertical wall and the fluid and therefore remains isothermal with temperature T_0. The conservation equations for mass, momentum and energy equations govern the velocity and temperature inside the hydrodynamic and thermal boundary layers

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial x}(\rho v) = 0 \qquad (3)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho g \sin\theta + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) (4)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right) (5)$$

Since the boundary layer thicknesses are typically very small, the following inequalities are known to apply,

$u \gg v$ and

$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}$ for velocity boundary layer, and

$\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x}$ for thermal boundary layer.

That is, the velocity component in the direction along the surface is much larger than that normal to the surface, and velocity and temperature gradients normal to the surface are much larger than those along the surface. The normal stress in the streamwise direction is negligible, and the single relevant shear stress component reduces to

$$\tau_{xy} = \tau_{yx} = \mu\frac{\partial u}{\partial y}$$

Upon using these assumptions, streamwise diffusion of streamwise momentum and heat is neglected in equations (4) and (5).

The boundary conditions for the above governing equations (3)-(5) are

$$u = v = 0, T = T_w \ at\ y = 0 (6a)$$

$$u(x, y) \to U(x)\ as\ y \to \delta(x) (6b)$$

$$T(x, y) \to T_0\ as\ y \to \delta_T (6c)$$

The coupled governing equations (3)–(5) with boundary conditions (6) completely define the flow system, which is identical to a compressible boundary layer problem, except that heat generation by viscous dissipation $\mu\left(\frac{\partial u}{\partial y}\right)^2$ is assumed to be negligible and the $u\frac{\partial p}{\partial x}$ is absent in equation (5) since in the present problem incompressible fluid is considered. However, the governing equations are similar in form with those obtained by Andersson et al. [2] who studied with incompressible fluid, water. Three dependent variables evolve in the system such as *u*, *v*, and *T*. For fluids with constant physical properties, the system reduces to the problem considered by Andersson. [**19**], a Falkner-Skan-type of similarity transformation is enough for the transformation of governing PDEs into a set of ODEs. However, in the present problem, ρ , μ , κ and C_p are temperature dependent, and must be known to solve the problem with given boundary conditions. The Falkner-Skan-type of transformation is no longer applicable, and a new similarity transformation, Howarth-Dorodnitsyn transformation is used to render the problem. Although the Howarth-Dorodnitsyn transformation was used in compressible boundary layer theory, recently Andersson et al. [2] used the same transformation for incompressible fluid. Following Andersson et al. [Reference], the Howarth-Dorodnitsyn transformation is used in this study for incompressible fluids with variable physical properties. Let us first define a stream function(x, y):

$$\rho u = \frac{\partial \psi}{\partial y}; -\rho v = \frac{\partial \psi}{\partial x} \qquad (7)$$

such that mass conservation (3) is automatically satisfied. The similarity variable η and the new dependent variables f and Θ are defined as:

$$\eta = (3U|4\nu_0 x)^{\frac{1}{2}} \int_0^y (\rho/\rho_{0)dy} \qquad (8)$$

$$\Psi(x, y) = \rho_0 \left({}^{4U\nu_0 x}/_3\right)^{\frac{1}{2}} f(\eta) \qquad (9)$$

$$\Theta(\eta) = \frac{T(x,y) - T_0}{T_w - T_0} \qquad (10)$$

$$\text{Now, } \frac{\partial \Psi}{\partial y} = \rho\sqrt{2gx \sin\Theta}\, f'(\eta) \qquad (11)$$

$$\frac{\partial \Psi}{\partial x} = \left(-\frac{1}{4}\right)\rho y f'(\eta)\left(\frac{2gx\sin\theta}{x}\right)^{\frac{1}{2}} + \frac{\sqrt{3}}{2}\rho_0 f(\eta) x^{-\frac{1}{4}}\left(\sqrt{2gx\sin\theta}\,\nu_0\right)^{\frac{1}{2}} \quad (12)$$

Now putting all values in equation (4) we get,

$$\rho\left(U\frac{\partial U}{\partial x} + V\frac{\partial V}{\partial y}\right) = \frac{\partial \Psi}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial \Psi}{\partial y}\frac{1}{\rho}\right) - \frac{\partial \Psi}{\partial x}\frac{\partial}{\partial y}\left(\frac{\partial \Psi}{\partial y}\frac{1}{\rho}\right) = 2\rho g\sin\theta[f'^2 - \left(\frac{3}{2}\right)ff''] \quad (13)$$

$$\rho g\sin\theta + \frac{\partial}{\partial y}\left(\mu\frac{\partial U}{\partial y}\right) = \rho g\sin\theta[\,1 + \left(\frac{3}{2}\right)f'''(\eta)\frac{\mu}{\mu_0}\frac{\rho}{\rho_0}] \quad (14)$$

Now combining equation (13) & (14) we will get,

$$[\frac{\rho\mu}{\rho_0\mu_0}f'']' + ff'' + \left(\frac{2}{3}\right)(\sin\theta - f'^2) = 0$$

Again putting all values in equation (5) we get,

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right)$$

$$\frac{3}{2\sqrt{2}}C_p\rho f(\eta)\theta'(\eta)x^{\frac{1}{2}}\sqrt{g\sin\theta}\left(T_w - T_0\right) = \kappa\left(T_w - T_0\right)\frac{\rho^2}{\rho_0^2}x^{-\frac{1}{2}}\Theta''(\eta)\frac{3\sqrt{(g\sin\theta)}}{2\sqrt{2}\nu_0}$$

The coupled boundary layer problem (3)-(6) now transforms into the following BVP:

$$[\frac{\rho\mu}{\rho_0\mu_0}f'']' + ff''\left(\frac{2}{3}\right)(\sin\theta - f'^2) = 0 \quad (15)$$

$$\left(\frac{\rho\kappa}{\rho_0\kappa_0}\theta'\right)' + \frac{C_p}{C_{p_0}}P_{r_0}f\theta' = 0 \quad (16)$$

$f=f'=0$ and $\Theta=1$ at $\eta=0$, (17a)

$f' \rightarrow 1$ and $\Theta \rightarrow 0$ as $\eta \rightarrow \infty$, (17b)

the resulting equation of (15) and (16) with boundary conditions (17) are solved by MATLAB

3 NUMERICAL MODELING

Here transformed BVP are solved by MATLAB software. We use 'bvp4c' solver to solve this problem. Prior to solving this problem with bvp4c we must write the differential equation as a system of first-order ODE's as given below

Let $f = y_1$

$f' = y_2$

$f'' = y_3$

$\Theta = y_4$

$\Theta' = y_5$

Now from equation (15) we get,

$$\frac{\rho K}{\rho_0 K_0}y_3' + y_1 y_3 + \left(\frac{2}{3}\right)(\sin\theta - y_2^2) = 0$$

$$=> y_3' = \frac{\rho_0 K_0}{\rho K}[-y_1 y_3 - \left(\frac{2}{3}\right)(\sin\theta - y_2^2)]$$

From equation (16) we get,

$$\frac{\rho\kappa}{\rho_0\kappa_0}y_5' + \frac{C_p}{C_{p0}}Pr_0 y_1 y_5 = 0$$

$$=> y_5' = \frac{\rho_0\kappa_0}{\rho\kappa}[-\frac{C_p}{C_{p0}}Pr_0 y_1 y_5]$$

Empirical correlations for the physical properties Water

Here, we consider water at atmospheric pressure and adopt the empirical correlations suggested by Shang et al.[20]

$$\rho(T) = \left[-4.88*10^{-3}K^{-2}(T-273)^2 + 999.9\right]kgm^{-3} \quad (18)$$

$$\mu(T) = \exp\left[-1.6-1150KT^{-1} + \left(690KT^{-1}\right)^2\right]*10^{-3}kgm^{-1}s^{-1} \quad (19)$$

$$\kappa(T) = [-8.01*10^{-6}K^{-2}(T-273K)^2 + 1.94*10^{-3}K^{-1}(T-273) + 0.563]\ W(mk)^{-1} \quad (20)$$

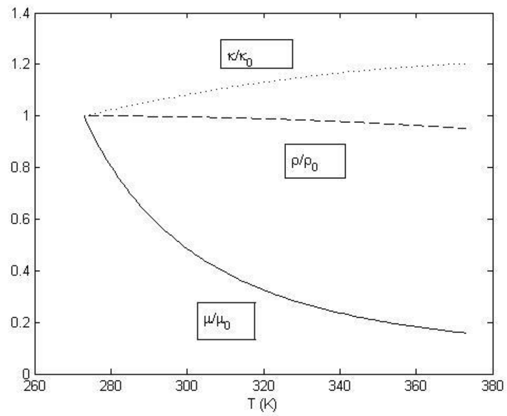

Fig. 2. Variation of ρ, μ and κ for water at atmospheric pressure

Here, T is taken as the absolute temperature in Kelvin. Where, $\mu_0 = 1.787*10^{-3}kgm^{-1}s^{-1}$, $\rho_0 = 999.8kgm^{-3}$, $\kappa_0 = .563W(mK)^{-1}$, Cp=Cpo=$4.20*10^3$ $Jkg^{-1}K^{-1}$

It is readily seen that the relative variation of the density is practically negligible

4 RESULT& DISCUSSION

The temperature T in Eqs. (18)-(20) is therefore taken as $T = T_0 + \Delta T\Theta(\eta)$,where $\Delta T = \lambda(T_w - T_0)$ Here λ is a dimensionless parameter equal to unity, provided that λ =0 is essentially gives constant physical properties, and λ <1 is not considered in this study since $T_w > T_o$. In general, however, the property relation takes the form

$$\rho = \rho(\Theta; T_0, \Delta T);\ \mu = \mu(\Theta; T_0, \Delta T);\ \kappa = \kappa(\Theta; T_0, \Delta T) \quad (21)$$

The boundary value problem is thus a three parameter problem of which the solution depends on T_0 and ΔT both with the Prandtl number Pr_0. For $\Delta T = 0$ the solution equivalent to single parameter problem in Pr_0.

In Fig. 3 and 4 some characteristic profile are shown. Here inflow temperature $\Delta T = 20^oC$ in all cases. From fig. 3 it can be observed that velocity f' is significantly changes by the temperature. Though viscosity reduced with T, it consequent that the thickness of the hydrodynamic boundary layer decrease with ΔT. When the wall temperature is lower than the inflow temperature then adjacent water release heat and its viscosity is increased. In fig. 3 the value of f' at η=1 for $\Delta T = -20^oC$ is 0.61 and the result given in Andersson is 0.61. They are approximately same as present value.

Table 1. Comparison between the present method (boundary value problem) and Andersson's work [2] (initial value problem) for the velocity profile $f'(\eta = 1)$ for different ΔT

ΔT	Present study	Andersson[2] value
-20	0.61	0.61
0	0.70	0.70
60	0.89	0.89

It is seen that thesis value are exactly the same with the values of Andersson's study. It shows the correctness of the method, and indicates that the coding for boundary value problem is right.

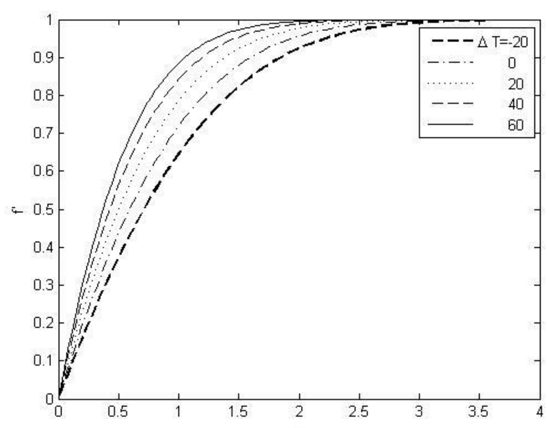

Fig. 3. Characteristic velocity profile $f'(\eta)$ for different ΔT

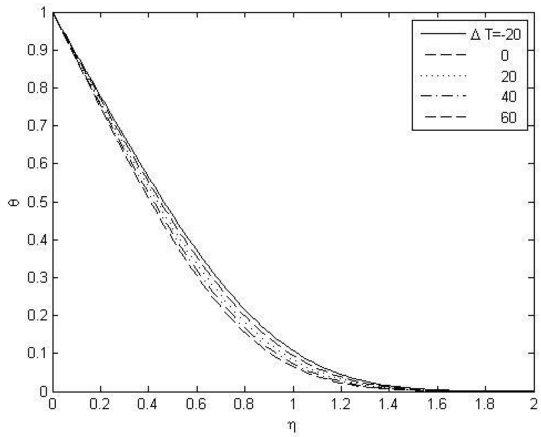

Fig. 4. Characteristic temperature profile $\Theta(\eta)$ for different ΔT

In Fig. 4 it is observed that the thickness of the thermal boundary layer δ_T is less than half of the hydrodynamic boundary layer thickness δ. In this figure all the temperature is remain constant after η=2. Here temperature profile is closer than velocity profile for same ΔT.

In Fig. 5 and 6 velocity and temperature profile has been observed for different Prandtlnumber Pr_0. These figures are has the similar pattern as figure 3 and 4 and shows same characteristic. Here all values are more same with Fig 3 and 4. In Fig. 5 velocity has higher value for Prandtl number. The value at η=1 for Pr_0=13.26 is 0.7.

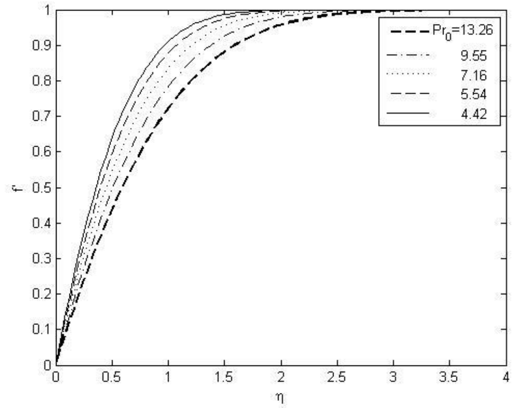

Fig. 5. Characteristic velocity profile $f'(\eta)$ for different Pr_0.

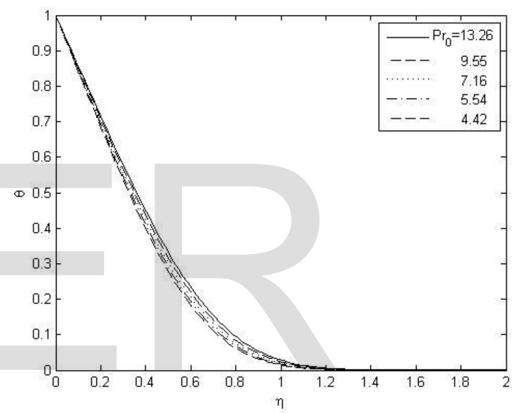

Fig. 6. Characteristic temperature profile $\Theta(\eta)$ for different Pr_0

In Fig. 7 and 8 wall shear is observed for different ΔT and Pr_0 respectively. If ΔT and Pr_0 increase then wall shear will be increase. All values are consistent at a same point before that these values intersect a point before reaching that point.

In Fig. 9 it has been observed that wall shear stress $f''(0)$ for different ΔT. For higher ΔT wall shear stress is also higher. Here $\Delta T = 20^oC$ and $\Delta T = 60^oC$. Wall shear stress is monotonically increasing with the increase of ΔT. Andersson has the same value for ΔT= 0 for $\Delta T = 20^oC$, so the two lines intersect with each other. In this thesis the value for ΔT= 0 are not same. The curve has lower slopes than the Andersson curves. At ΔT= 0 the present thesis then reduces to the constant physical properties and considered by Andersson and f"(0) becomes equal to 1.03890 but here obtained value is 1.0567 which is slightly differ from Andersson value.

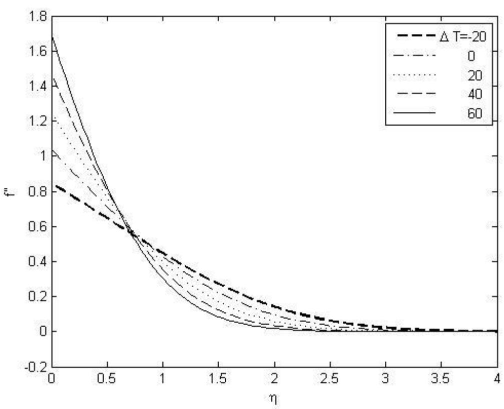

Fig. 7. Variation of shear stress f'' with η for different ΔT

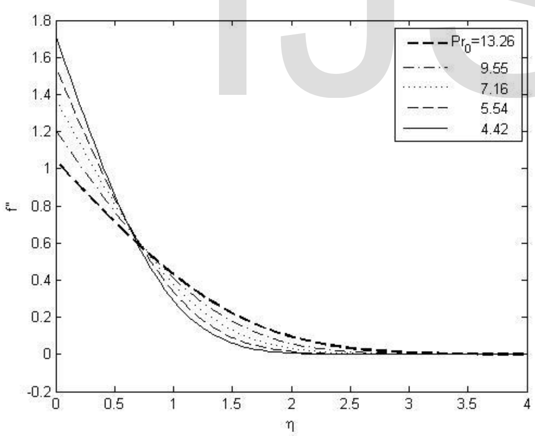

Fig. 8. Variation of shear stress f'' with η for different Pr_0

In Fig. 12 it has been seen that the heat transfer rate at the wall is directly related to the wall temperature gradient $\Theta^{'}(0)$. The dimensionless temperature gradient is gradually increase with the increase of ΔT. The value of $\Theta'(0)$ is linearly increasing with the increase of ΔT. Thesis figure are more similar with Andersson figure. When the wall temperature is greater than inflow temperature then heat will be transferred from the wall to the liquid film and vice versa. This is a consequence of the thinning of the thermal boundary layer with increasing temperature, as shown in Fig. 4. As suggested above, the thinningof the thermal boundary layer is directly associated with the thinning of the hydrodynamic boundary layer.

Paper is performed by boundary value problem. Some initial guess has to take for the solution of ODEs. The difference between Andersson value and this thesis value may be occurred for guessing initial value. There might be some error for guessing those initial values. The pattaren of the figures are almost same but values are slightly differing from thesis value.

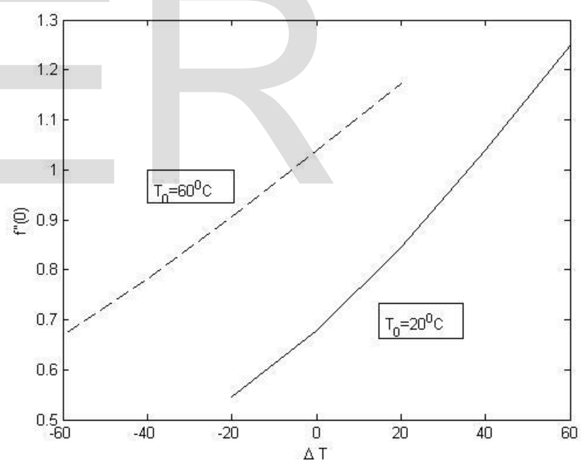

Fig. 9. Variation of wall shear stress f"(0) with ΔT for T_0= 20⁰C and T_0=60⁰C

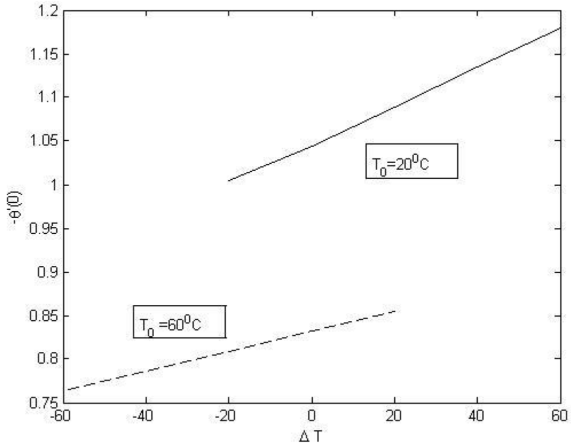

Fig. 10. Variation of wall shear stress Θ'(0) with ΔT for T_0= 20^0C and T_0=60^0C

These important film characteristics observed above determine the local skin-friction coefficient and local Nusselt number. The formulas are given below:

$$C_f \equiv \frac{\tau_w}{\left(\frac{1}{2}\right)\rho_0 U^2} = \frac{\rho_w \mu_w}{\rho_0 \mu_0} . 3^{\frac{1}{2}} . Re_x^{-\frac{1}{2}} . f''(0) \qquad (22)$$

$$Nu_x \equiv -\frac{\rho_w}{\rho_0}\left(\frac{3}{4}\right)^{\frac{1}{2}} . Re_x^{\frac{1}{2}} . \Theta'(0) \qquad (23)$$

Here considering x=1, U will be obtained from equation (2) and $\theta = 90^0$

Using equation (22) and (23) following values has been calculated:

Table 2. Skin-friction coefficient and local Nusselt number for different ΔT

No. obs	T_0	ΔT	C_f	Nu_x
1	20^0C	-20	9.64*10^(-4)	1512
2		0	5.03*10^(-4)	2059
3		20	3.11*10^(-4)	2623
4		40	2.16*10^(-4)	3174
5		60	1.62*10^(-4)	3681

From the above table it is clearly seen that the value of Nusselt number is increasing with the increase of $\Delta T$and skin friction is decreasing with the increase ofΔT.

5 CONCLUSION

The result of this paper has been presented in the previous chapter. The similarity transformation (8)-(10) has been devised to enable a proper investigation of the influence of variable physical properties on a gravity driven film flow along an inclined wall which is same as Andersson[2]. Based on those results some conclusions can be drawn. Film flow acceleration and heat transfer rate is increased with the increase of ΔT. Thermal and hydrodynamic boundary layer becomes thinner with the increase of ΔT. Skin friction coefficient is decreasing with the increase of ΔT and Nusselt no. is increasing with the increase of ΔT.

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REFERENCES

[1] H. I. Andersson, B. A. Pettersson, and B. S. Dandapat, "Combined forcedand natural convection in laminar film flow," Wärme- und Stoffübertrag.29, 399 1994.

[2] H. I. Andersson, B. Santra, B. S. Dandapat," Gravity-driven film flow with variable physical properties." 2006

[3] L. Prandtl, ¨ UbDieFlussigkeitsbewegungbeiSehrKleinerReibung, Proc. ThirdInt. Math. Koug., Heidelberg, 1904

[4] Shang Deyi," Free convection film flows and heat transfer." Springer-Verlag Berlin Heidelberg 2006

[5] A. M.Clausing and S. N. Kempka, The influences of property variations onnatural convection from vertical surfaces, J. Heat Transfer 103, pp. 609–612,1981

[6] T. Fujii et al., Experiments on natural convection heat transfer from theouter surface of a vertical cylinder to liquids, Int. J. Heat Mass Transfer 13,pp. 753–787, 1970

[7] T. T. Hara, The free-convection flow about a heated vertical plate in air, Trans.Jpn. Soc. Mech. Eng. 20, pp. 517–520, 1954

[8] A. A. Tataev, Heat exchange in condition of free laminar movement of gas withvariable viscosity at a vertical wall, Zh. Tekh. Fiz. 26, pp. 2714–2719, 1956

[9] E. M. Sparrow and J. L. Gregg, The variable fluid-property problem in freeconvection, Trans. ASME, 80, pp. 879–886, 1958

[10] D. D. Gray and A. Giogini, The validity of the Boussinesq approximation forliquids and gases, Int. J. Heat Mass Transfer 19, pp. 545–551, 1977

[11] R. A. Mashelker and V. V. Chavan, Solid dissolution in falling films of non-Newtonian liquids, J. Chem. Jpn 6, pp. 160–167, 1973

[12] V. C. Van der Mast, S.M. Read, and L.A. Bromley, Boiling of natural sea waterin falling film evaporators, Disalinations 18, pp. 71–94, 1976

[13] I. Pop, T. Watanabe, and H. Komishi, Gravity-deriven laminar film flow alonga vertical wall with surface mass transfer, Int. Commun. Heat Mass Transfer23(5), pp. 687–695, 1996

[14] T. M. T. Yang and D. W. Yarbrough, A numerical study of the laminarflow of non-Newtonian fluids along a vertical wall, ASME J. Appl. Mech. 40,pp. 290–292, 1973

[15] T. M. T. Yang and D. W. Yarbrough, Laminar flow of non-Newtonian liquidfilms inside a vertical pipe, Rheol. Acta 19, pp. 432–436, 1980

[16] V. NarayanaMurthy and P. K. Sarma, A note on hydrodynamics entrance lengthof non-Newtonian laminar falling films, Chem. Eng. Ser. 32, pp. 566–567, 1977

[17] M. N. Tekic, D. Posarac, and D. A. Petrovic, A note on the entrance regionlengths of non-Newtonian laminar falling films. Chem. Eng. Ser. 41, pp. 3230–3232, 1986

[18] H. I. Andersson and F. Irgens, Hydrodynamic entrance length of non-Newtonianliquid films, Chem. Eng. Sci. 45, pp. 537–541, 1990

[19] H. I. Andersson, "Diffusion from a vertical wall into an accelerating falling liquid film," Int. J. Heat Mass Transfer **30**, 683(1987).

[28] D.-Y. Shang, B.-X. Wang, Y. Wang, and Y. Quan, "Study on liquid laminarfree convection with consideration of variable thermophysical properties,"Int. J. Heat Mass Transfer 36, 3411 1993